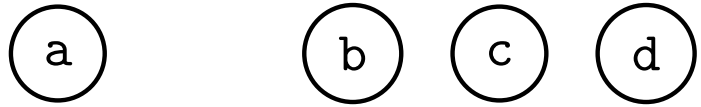
Optimal Shuffle Code with Permutation Instructions

Sebastian Buchwald, Manuel Mohr, Ignaz Rutter

Chair for Programming Paradigms & Chair for Algorithmics, Karlsruhe Institute of Technology (KIT)

Register Allocation and Shuffle Code

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c = 30;
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print(a, a, c, b, d);
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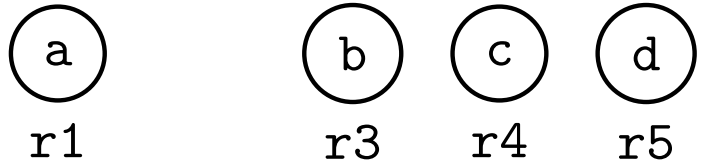

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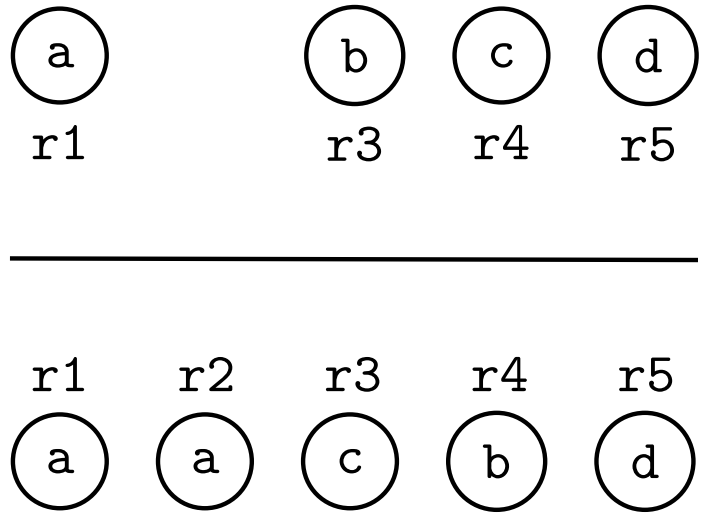

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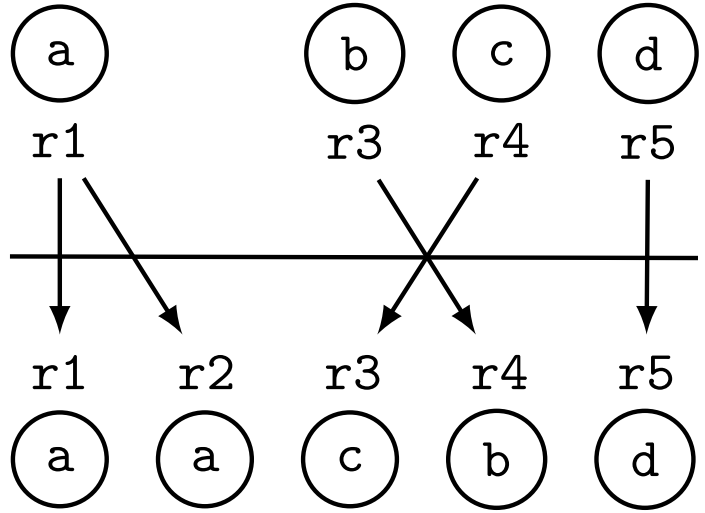

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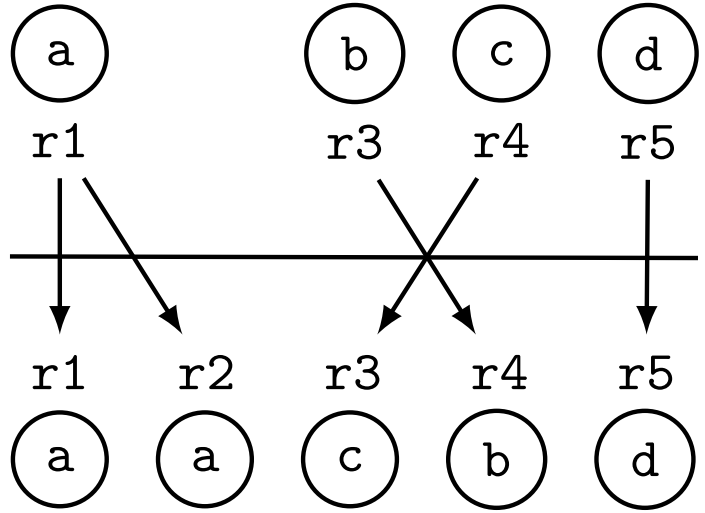
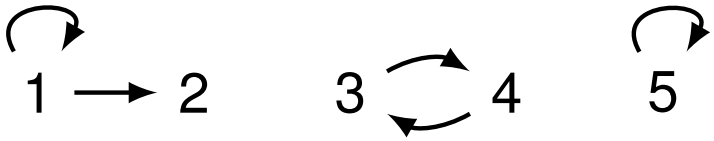


Register Transfer Graph (RTG)



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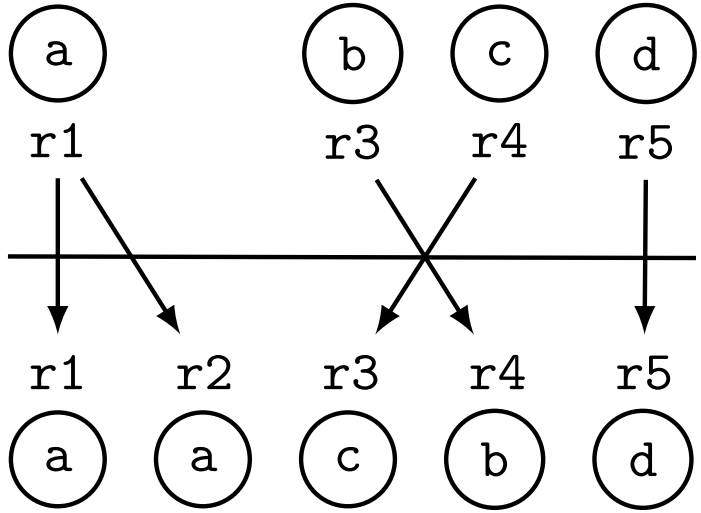

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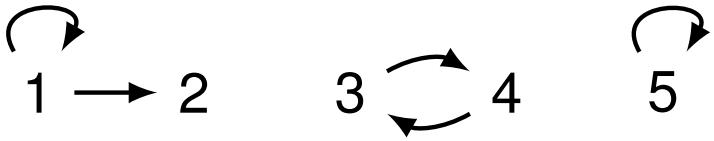


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swap r3, r4
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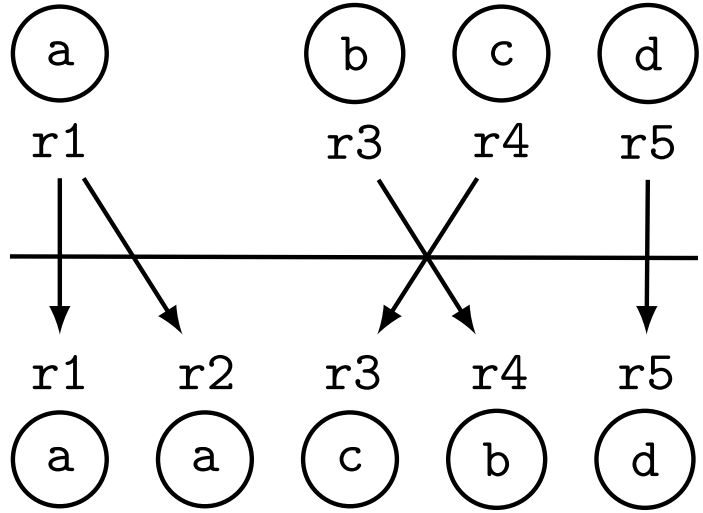

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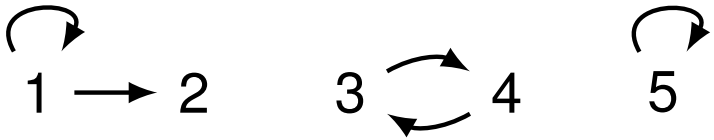


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Shuffle Code Generation Problem

Find a shortest shuffle code that **implements** a given RTG.

Permutation Instructions

- For large RTGs, implementations can get lengthy

$\Rightarrow$ Proposed more powerful permutation instructions [Mohr et al. 2013]

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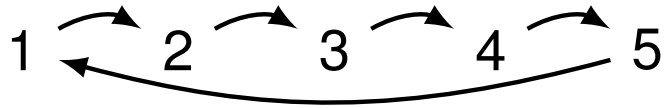


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permi5 r1, r2, r3, r4, r5
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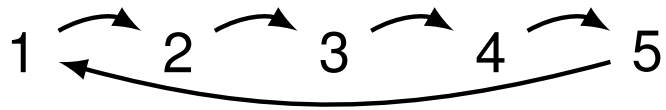


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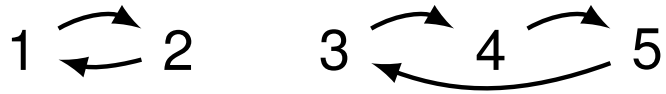


```
permi23 r1, r2, r3, r4, r5
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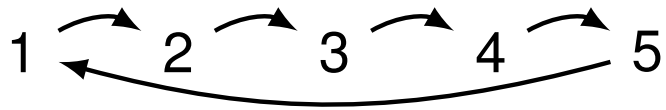


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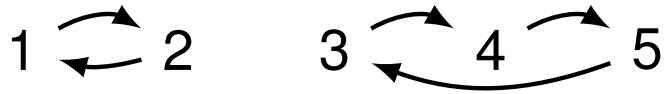


```
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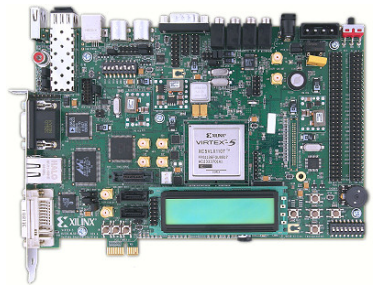
```

- Exists as FPGA-based prototype processor with modified compiler [Mohr et al. 2013]
  - Improves performance in practice
  - Uses greedy heuristic



Shuffle Code with Permutation Instructions

Karlsruhe Institute of Technology

SCGP with permutation instructions

Given RTG, find a shortest shuffle code using `permi5`, `permi23` and `copy`.

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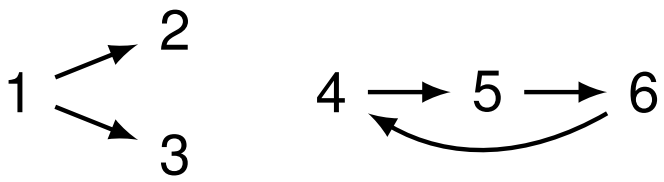

Naive implementation:

```
copy r1, r2
copy r1, r3
permi5 r4, r5, r6
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Shuffle Code with Permutation Instructions

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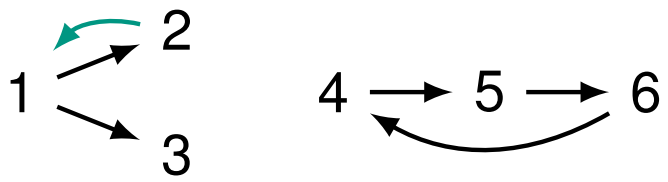

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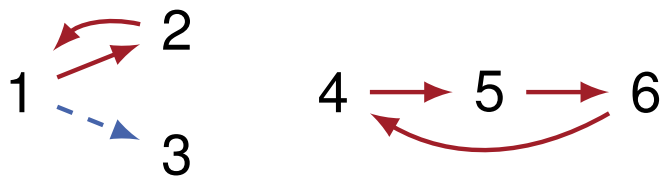

Optimal implementation:

```
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copy r2, r3
```

Shuffle Code with Permutation Instructions

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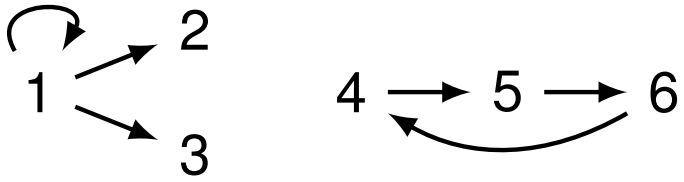

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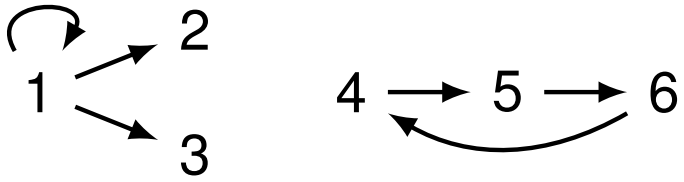

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Shuffle Code Generation as a Graph Problem

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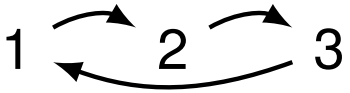
Effect of a **permutation**:

$$\pi \bullet (V, E) = (V, \{(\pi(u), v) \mid (u, v) \in E\})$$

$$[(1\ 2) \circ (2\ 3)] \bullet$$



$\xrightarrow{?}$

Shuffle Code Generation as a Graph Problem

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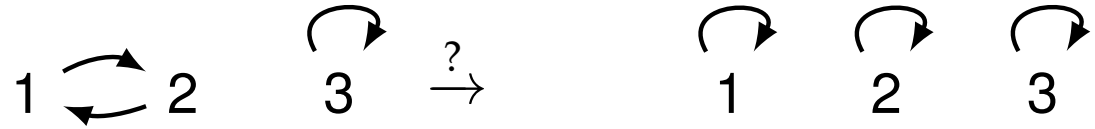
$$[(1\ 2)] \bullet$$ 

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- For permutation RTGs (PRTGs): make given PRTG trivial

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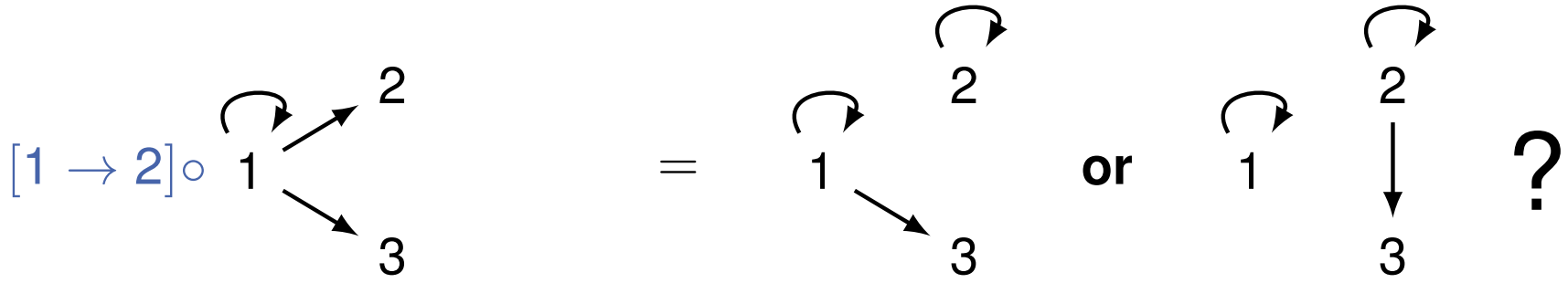
Effect of a **copy**:



- Copies not expressible in RTGs

Observations on Shuffle Code

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Copy $a \to b$ followed by transposition $\tau = (c\ d)$

rewrite to

Transposition $(c\ d)$ followed by copy $\tau(a) \to \tau(b)$

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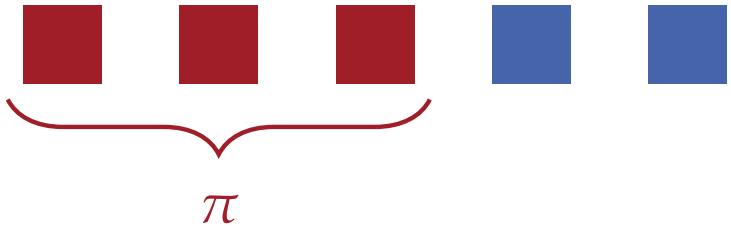
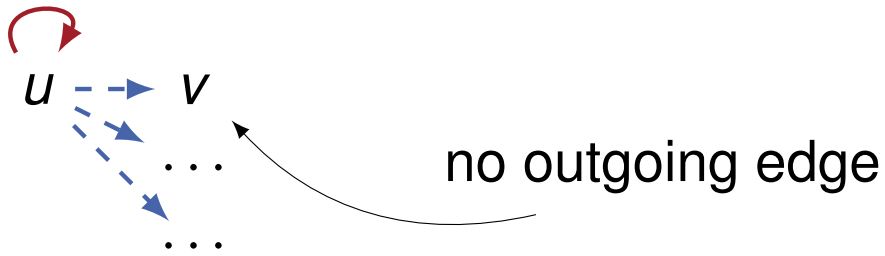


Can transform shuffle code such that in πG, for every copy $u \to v$:



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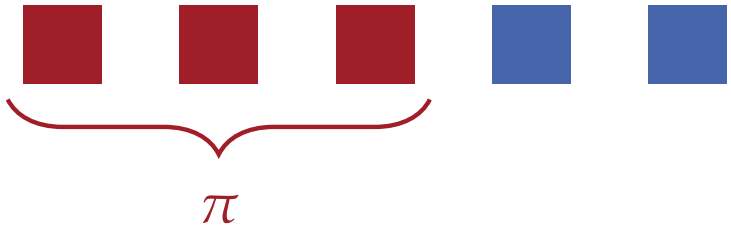

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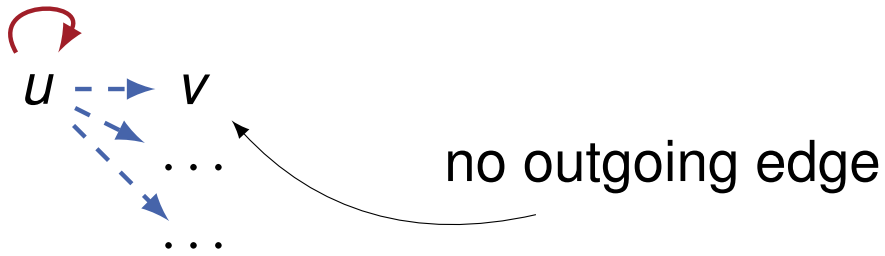

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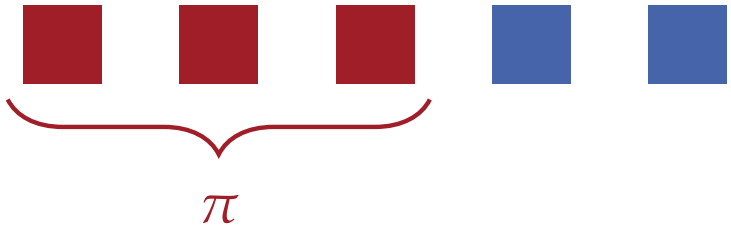

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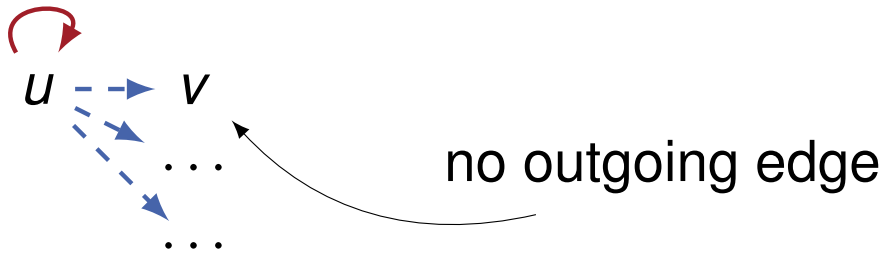

- All outgoing edges of u in πG are copies

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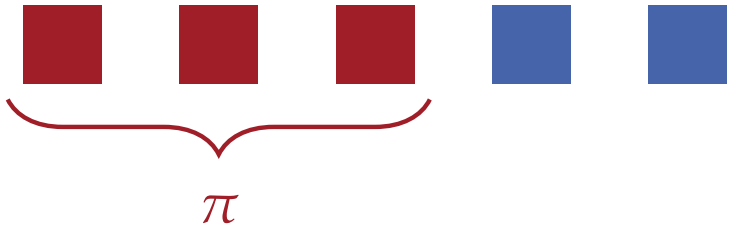

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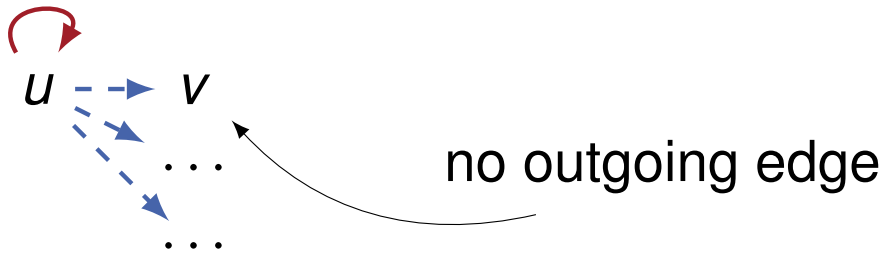

- All outgoing edges (except one) of u in πG are copies

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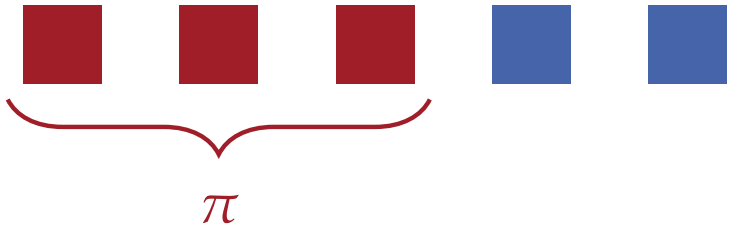

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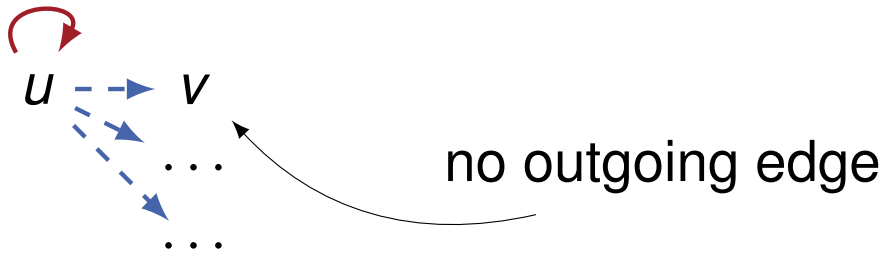

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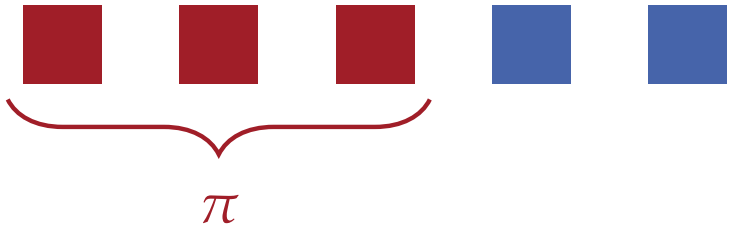

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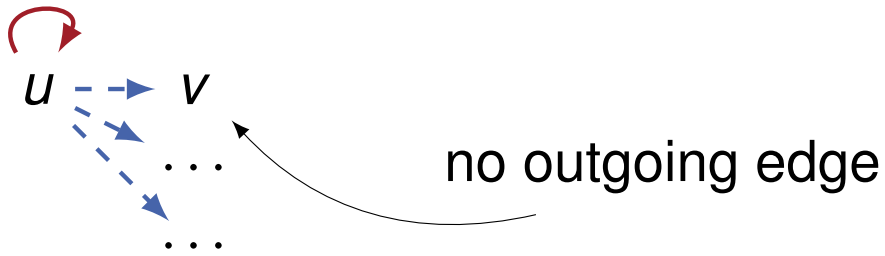

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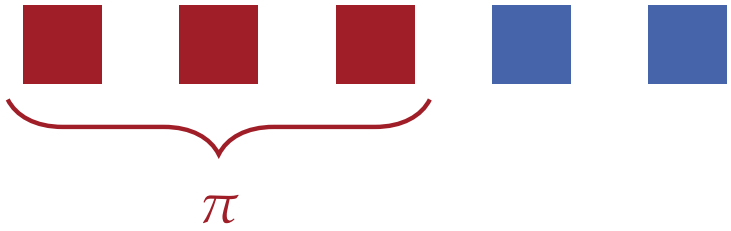

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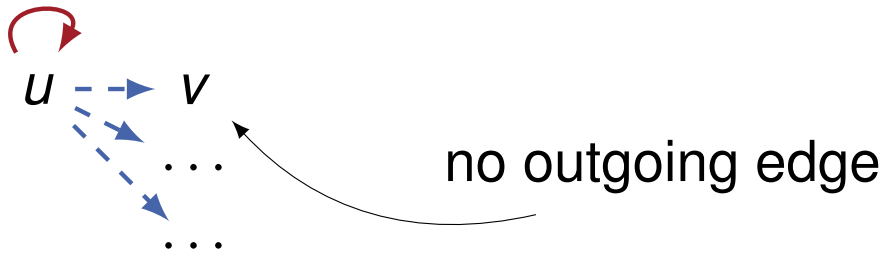

- All outgoing edges (except one) of u in πG are copies
- π permutes edge sources in G

$\Rightarrow$ All outgoing edges (except one) of u in G correspond to copies
$\to$ **copy set**

Strategy & Outline

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Step 1: Pick a copy set C.
(For each vertex, color one outgoing edge red, rest blue.)

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Step 3: Implement blue edges using copy operations.

Strategy & Outline

Choice of copy set is crucial:

Step 1: Pick a copy set C.
(For each vertex, color one outgoing edge red, rest blue.)

Step 2: Find an optimal shuffle code for the red RTG $G - C$.
(Has maximum out-degree 1.)

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(For each vertex, color one outgoing edge red, rest blue.)

Step 2: Find an optimal shuffle code for the red RTG $G - C$.
(Has maximum out-degree 1.)

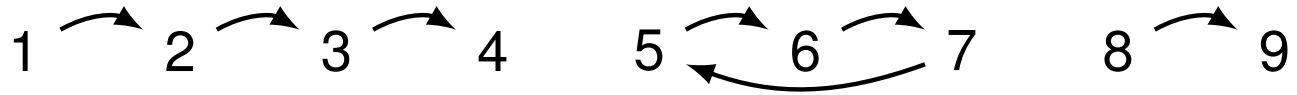
Step 3: Implement blue edges using copy operations.

Problem 1: Given copy set C, compute optimal shuffle code for $G - C$.

Problem 2: Find copy set C where $G - C$ requires fewest instructions.

The GREEDY Algorithm (adapted from [Mohr et al. 2013])

- Complete each directed path into directed cycle



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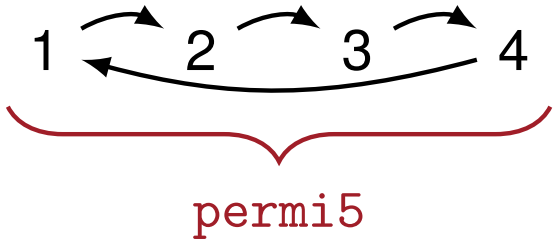

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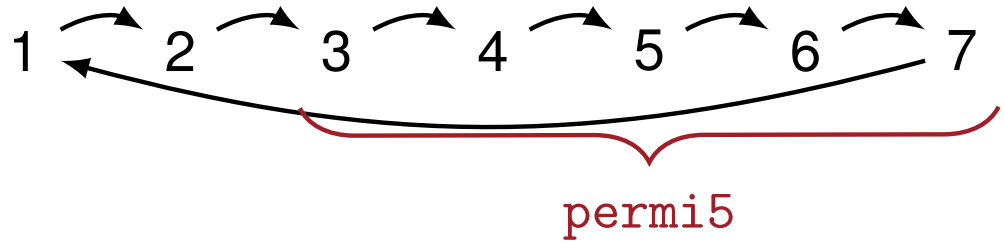

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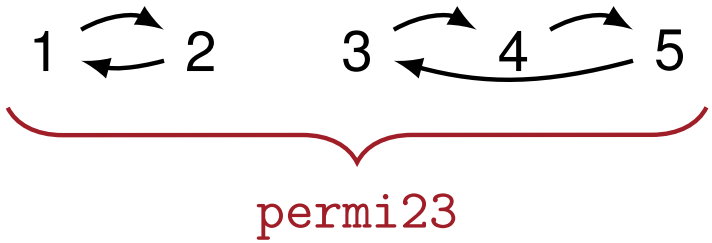

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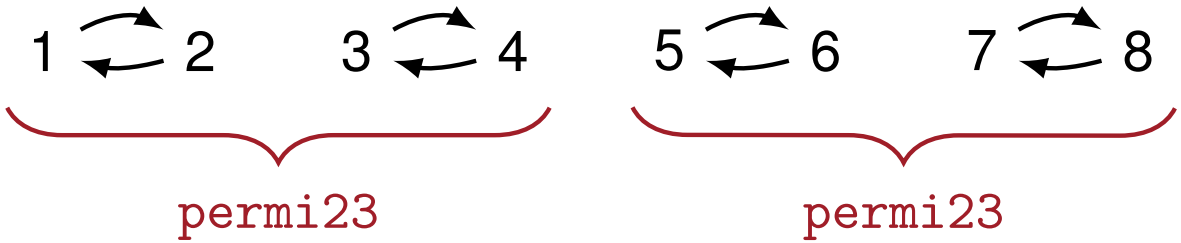

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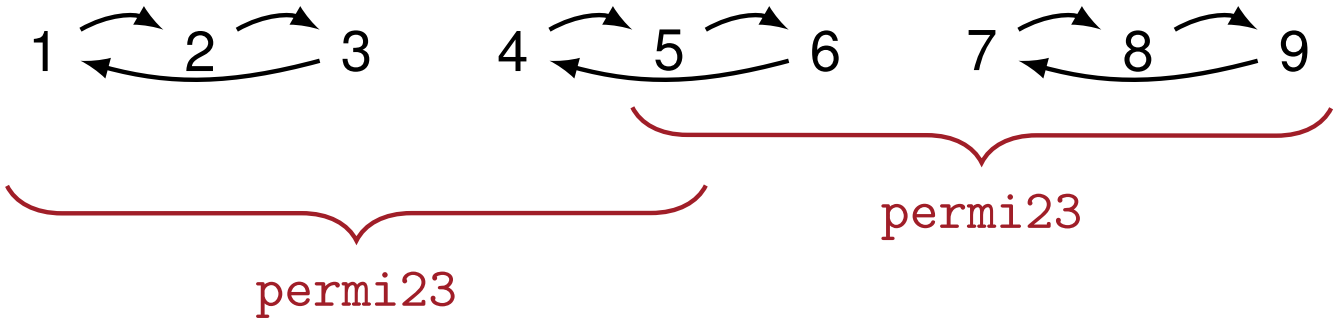

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Theorem

GREEDY is optimal for PRTGs.

Proof Outline

G

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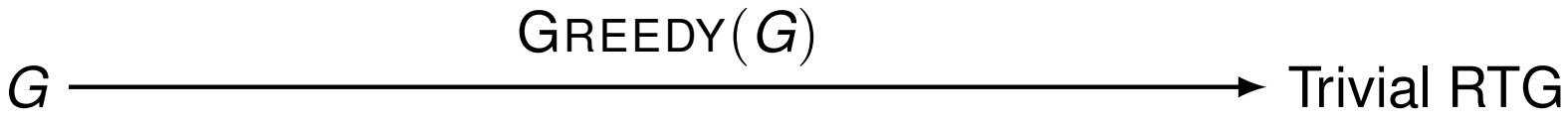

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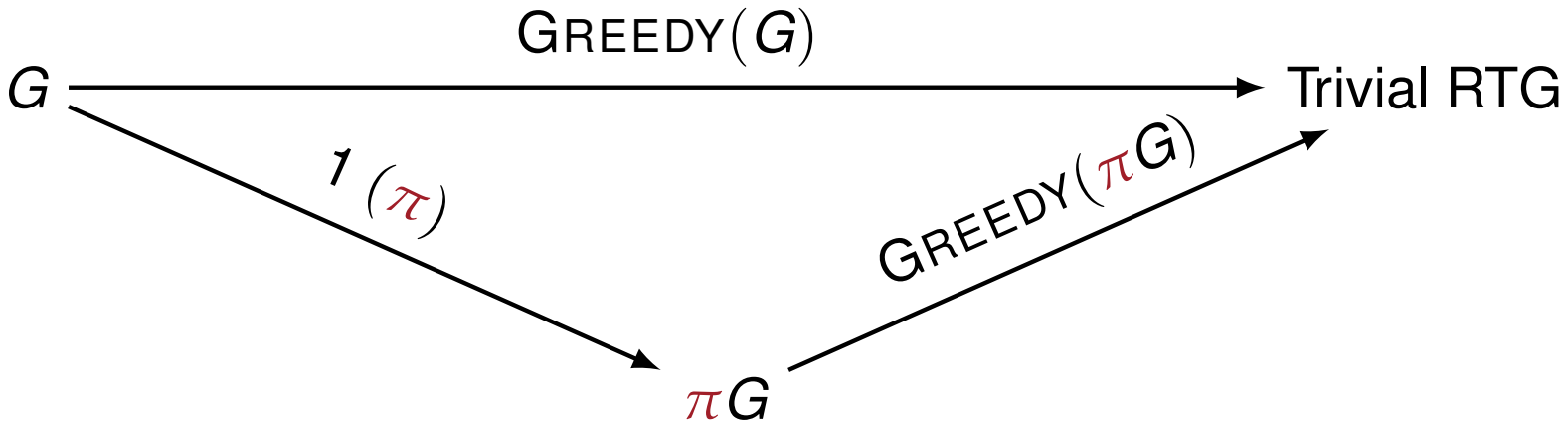

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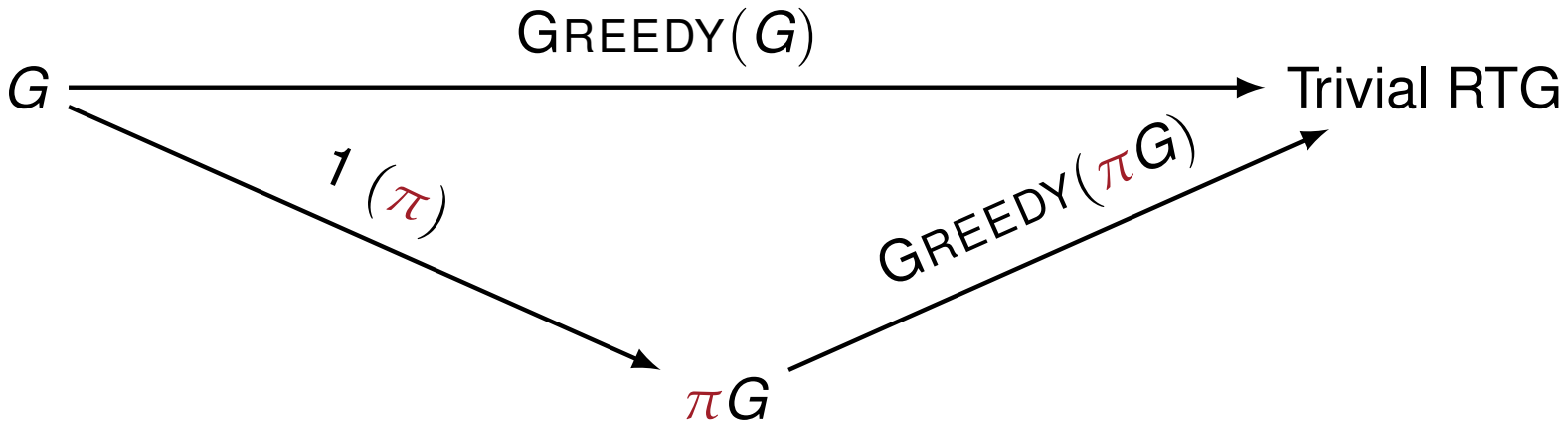
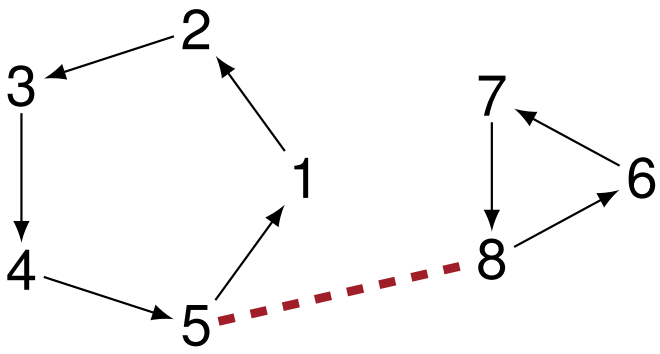


- Take arbitrary permutation instruction π
- **Show** that **always** $\textsc{Greedy}(G) \leq \textsc{Greedy}(\pi G) + 1$
  $\Leftrightarrow \textsc{Greedy}(G) - \textsc{Greedy}(\pi G) \leq 1$

Transpositions



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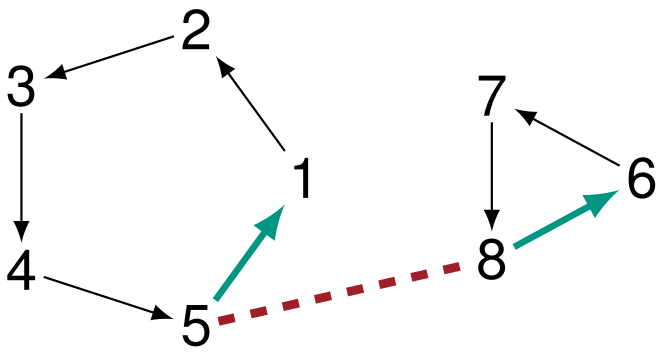

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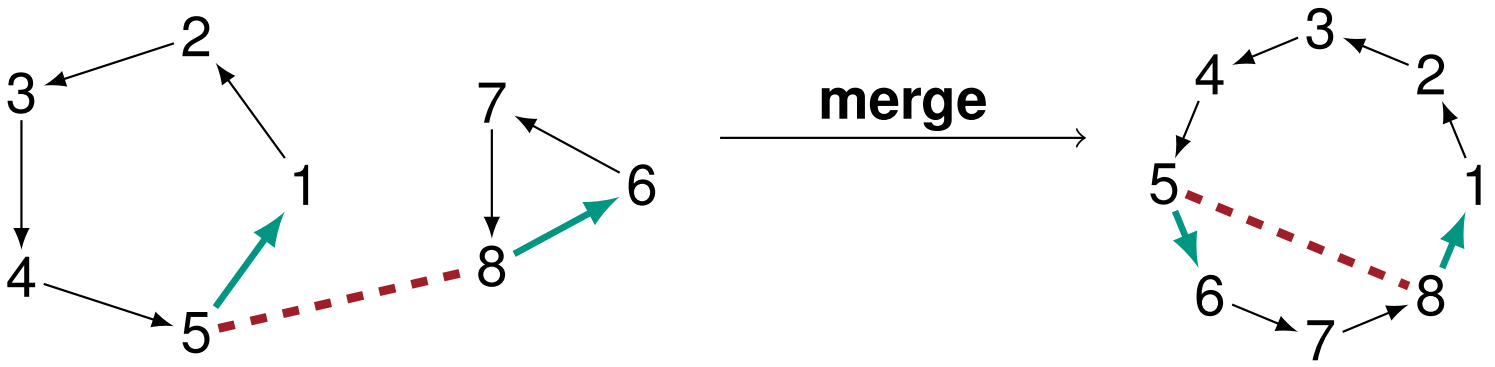

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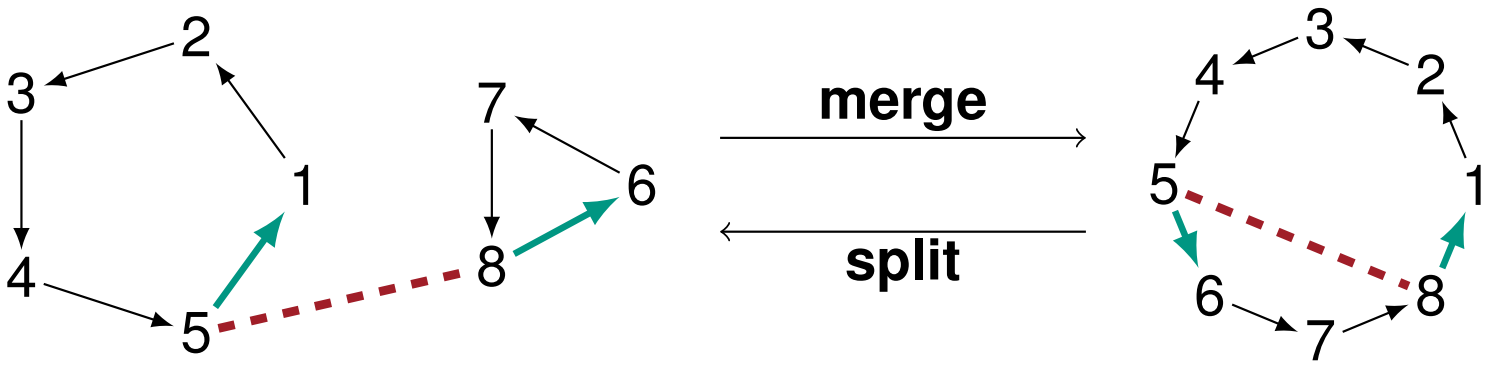

Merges

- Look at all possible signature changes $(\Delta_X, \Delta_2, \Delta_3)$

Merges

Cycle sizes modulo 4

	0	1	2	3
0	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$
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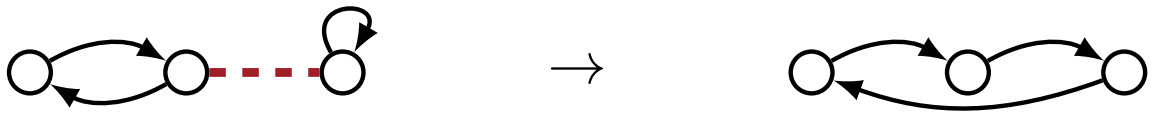

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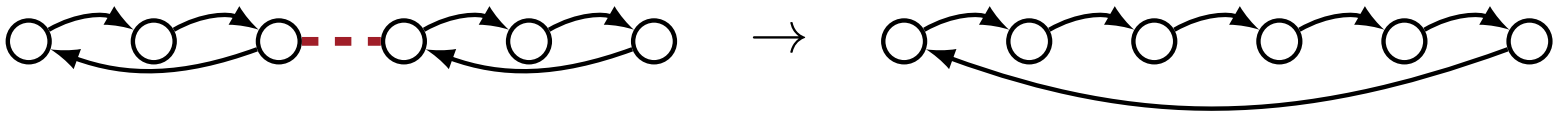

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0	0	0	0	0
1		½	0	½
2			0	0
3				½

$a_2 \geq a_3$

	0	1	2	3
0	0	0	0	0
1		⅓	⅓	⅓
2			⅓	0
3				0

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$\Rightarrow$ **Merges are never worthwhile!**

Splits

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GREEDY is optimal for PRTGs

GREEDY computes an optimal shuffle code for PRTGs in linear time.

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Given G, have to find copy set C, s.t. $\textsc{Greedy}(G - C)$ is minimal

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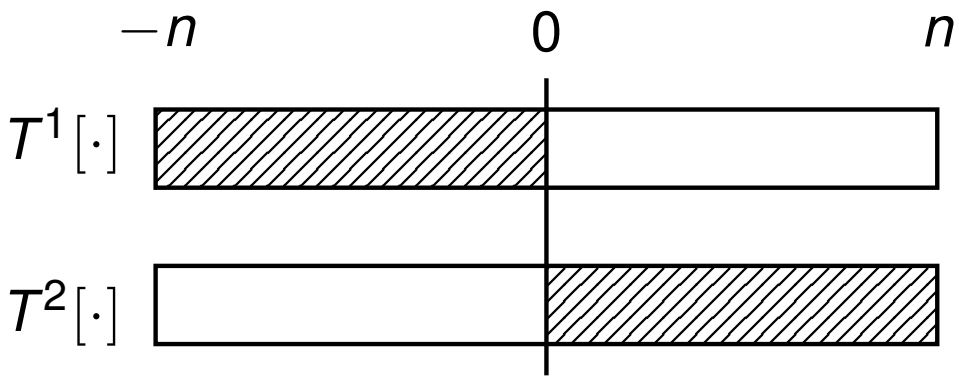
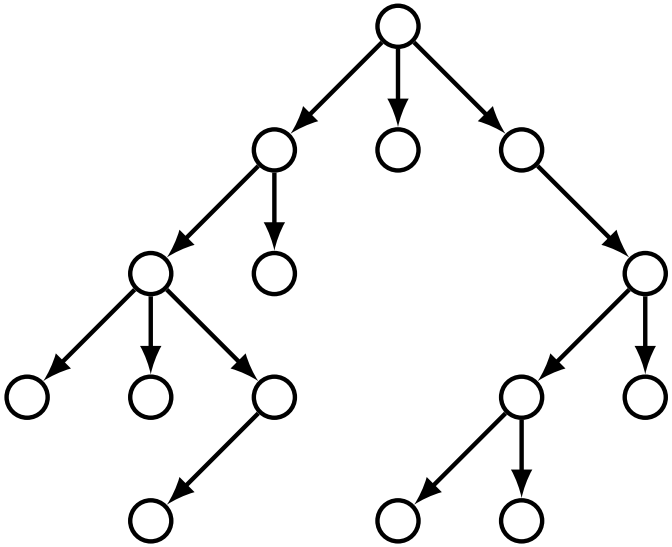
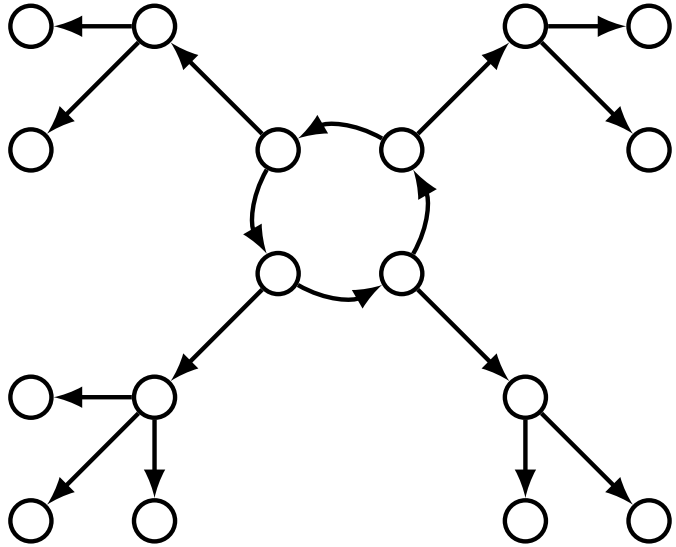


Computing Tables for Different RTG Shapes



Disconnected RTGs



$O(n^2)$

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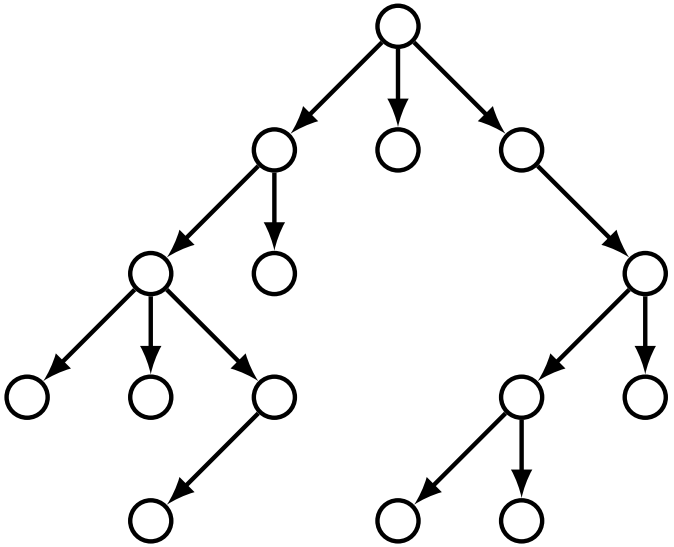
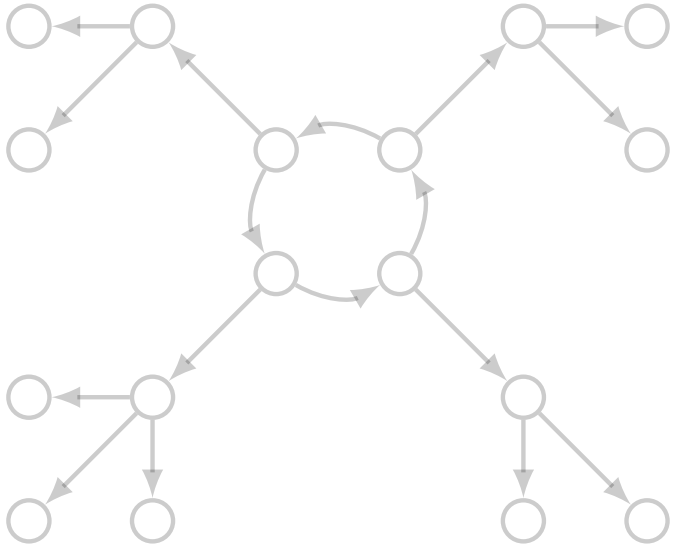
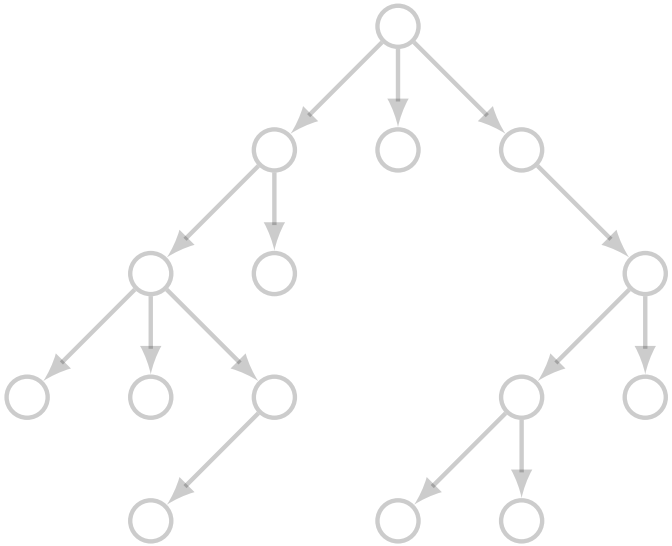
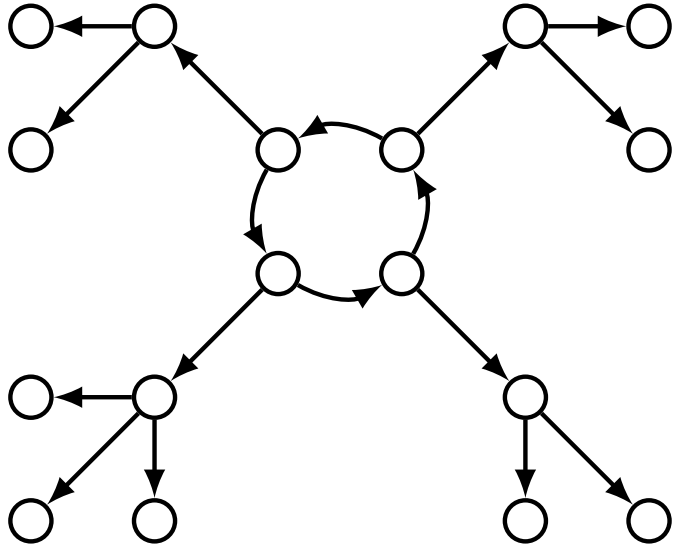




Disconnected RTGs	$O(n^2)$
Tree RTGs	$O(n^3)$

Computing Tables for Different RTG Shapes





Disconnected RTGs	$O(n^2)$
Tree RTGs	$O(n^3)$
Connected RTGs containing cycle	$O(n^4)$

Conclusion

Problem 1: Given copy set C, compute optimal shuffle code for $G - C$.

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Problem 2: Find copy set C s.t. $G - C$ requires fewest instructions.

- Used dynamic programming to solve optimally
- Runs in $O(n^4)$ time

Future work:

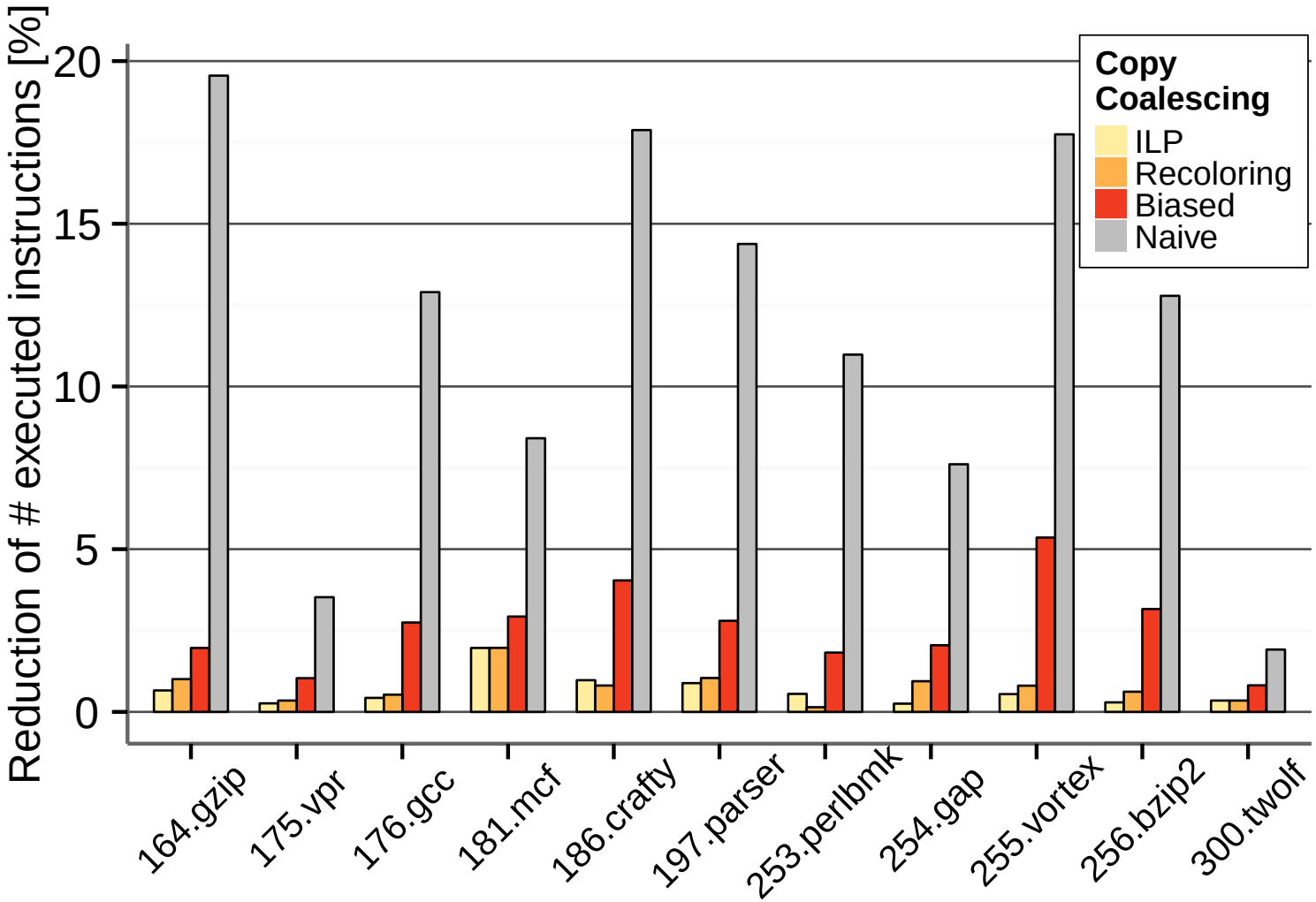
- Works for $k = 3, 4, 5, 6$, what about larger sizes?
- Problem NP-complete if permutation size is part of input

$\Rightarrow$ FPT-algorithm?

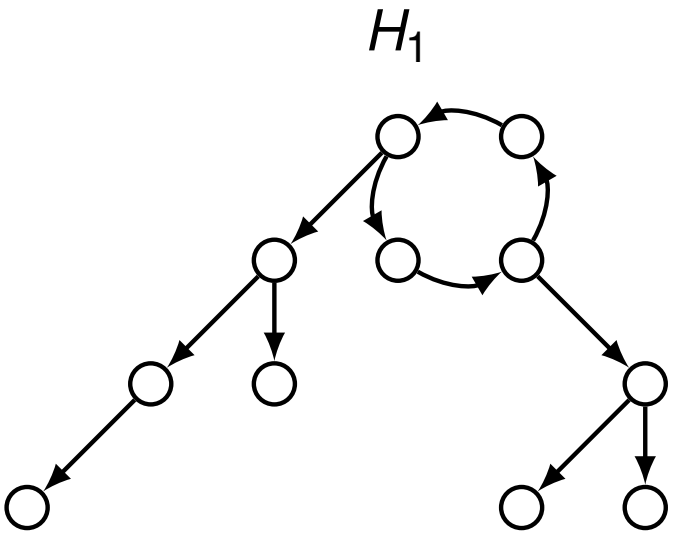
Karlsruhe Institute of Technology

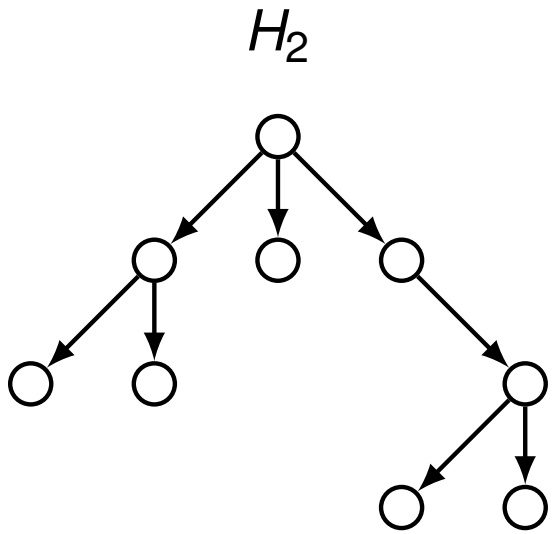
Backup Slides

Speedup Measurements

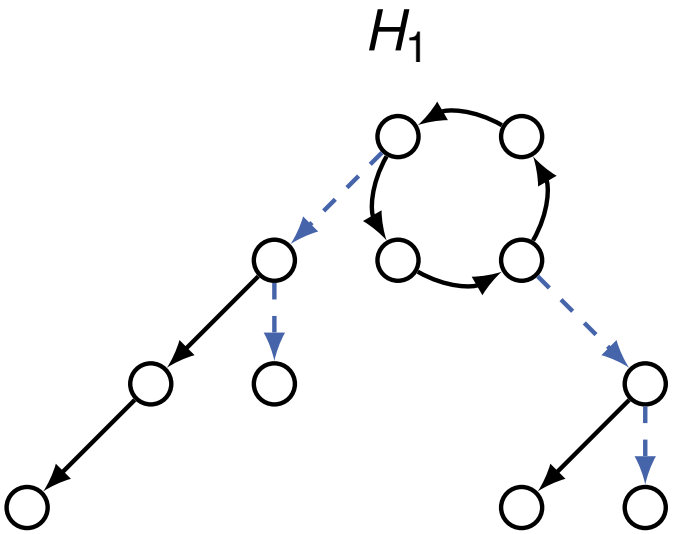


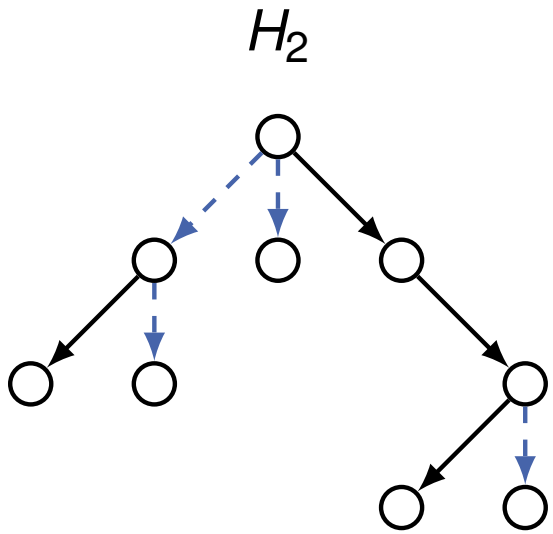
Disconnected RTGs



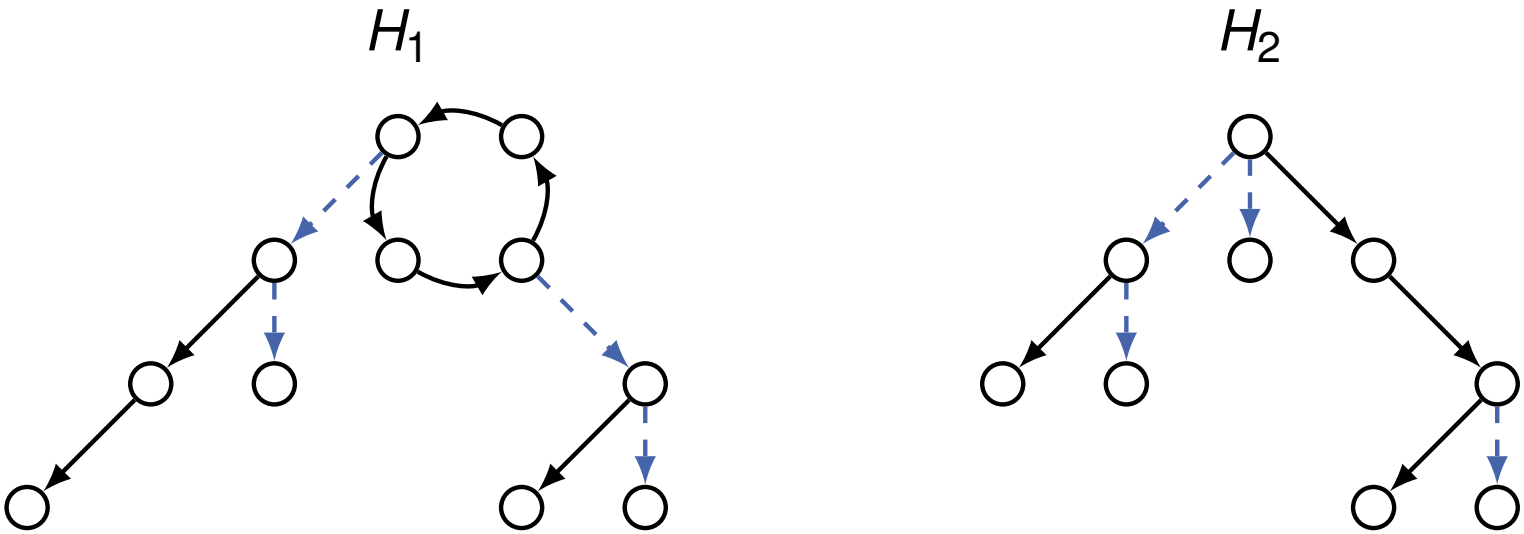


Disconnected RTGs



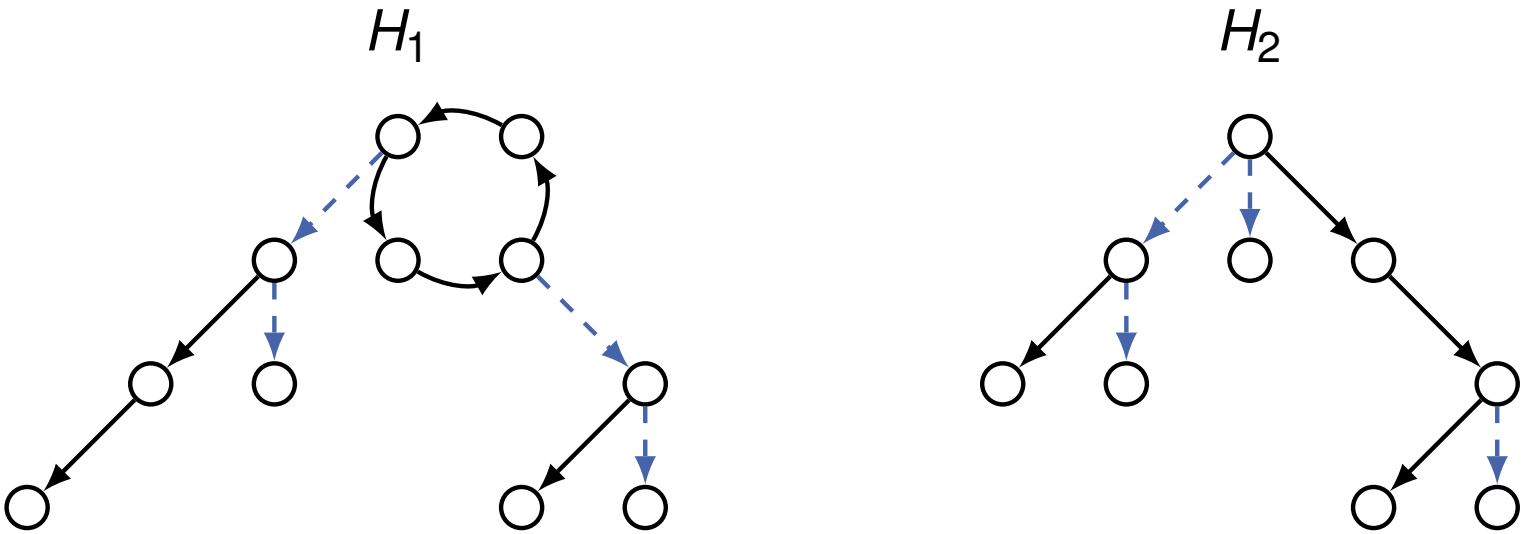


Disconnected RTGs



- $C := C_1 \cup C_2$
- $\text{cost}(G - C) = \text{cost}(H_1 - C_1) + \text{cost}(H_2 - C_2)$
- $\text{diff}(G - C) = \text{diff}(H_1 - C_1) + \text{diff}(H_2 - C_2)$

Disconnected RTGs

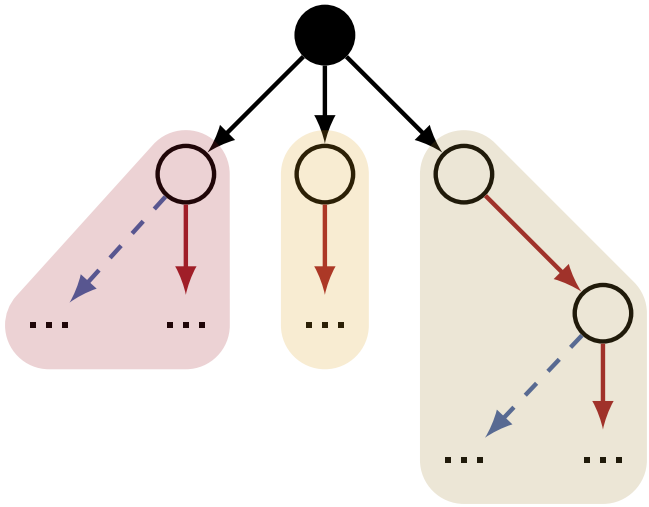


- $C := C_1 \cup C_2$
- $\text{cost}(G - C) = \text{cost}(H_1 - C_1) + \text{cost}(H_2 - C_2)$
- $\text{diff}(G - C) = \text{diff}(H_1 - C_1) + \text{diff}(H_2 - C_2)$

$\Rightarrow$ Can find optimal copy set C by trying all pairs C_1, C_2

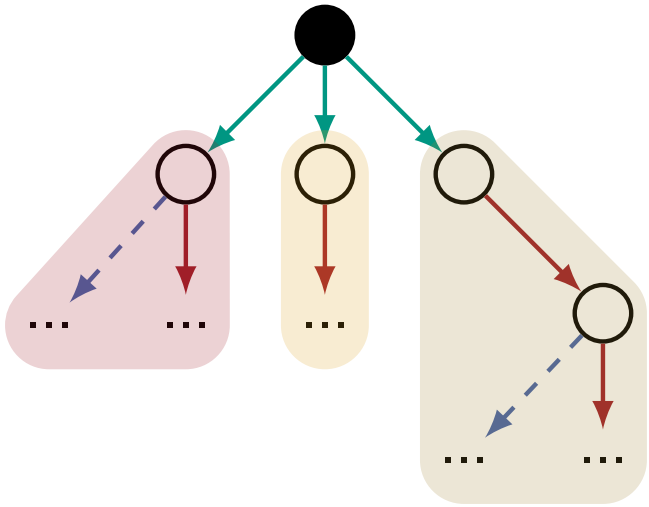
$\Rightarrow$ Can be computed in $O(n^2)$ time

Tree RTGs



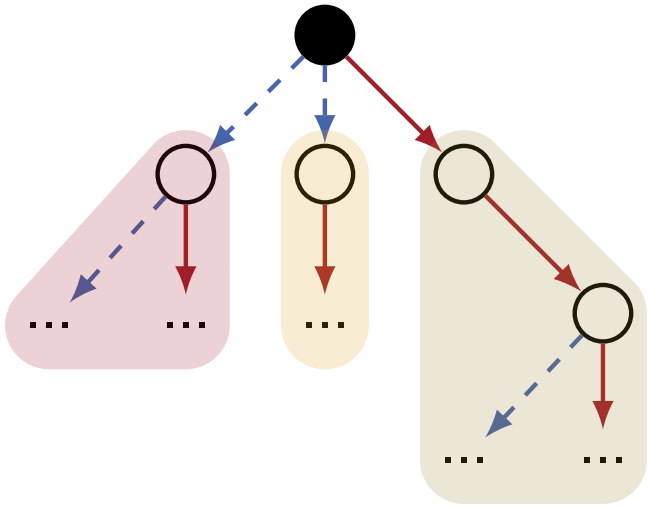
- Process tree RTGs in bottom-up fashion

Tree RTGs



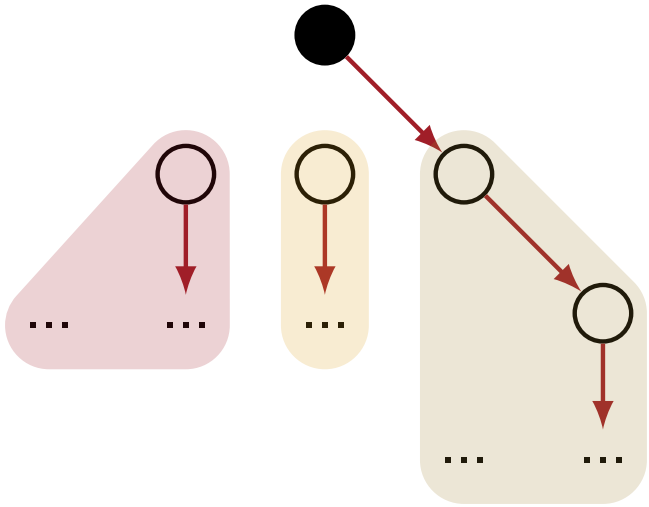
- Process tree RTGs in bottom-up fashion

Tree RTGs



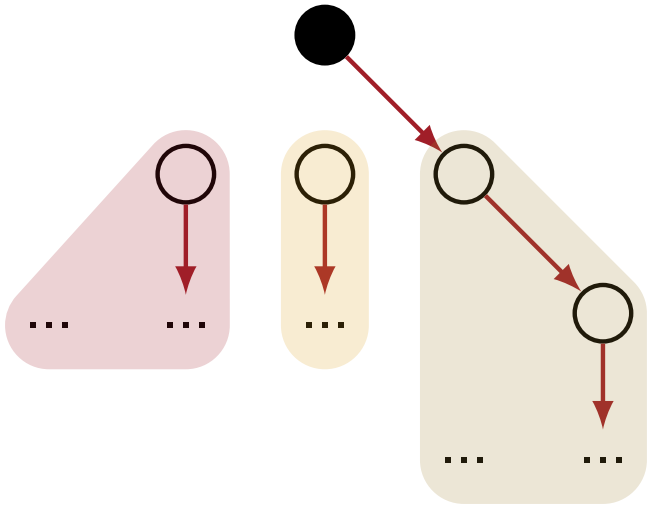
- Process tree RTGs in bottom-up fashion

Tree RTGs



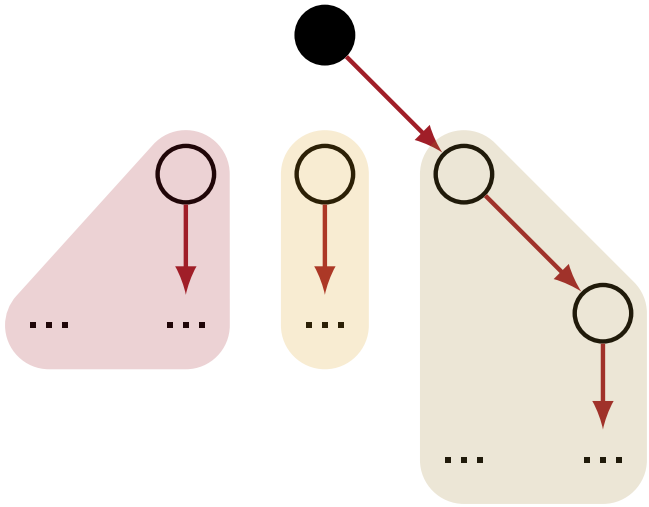
- Process tree RTGs in bottom-up fashion

Tree RTGs



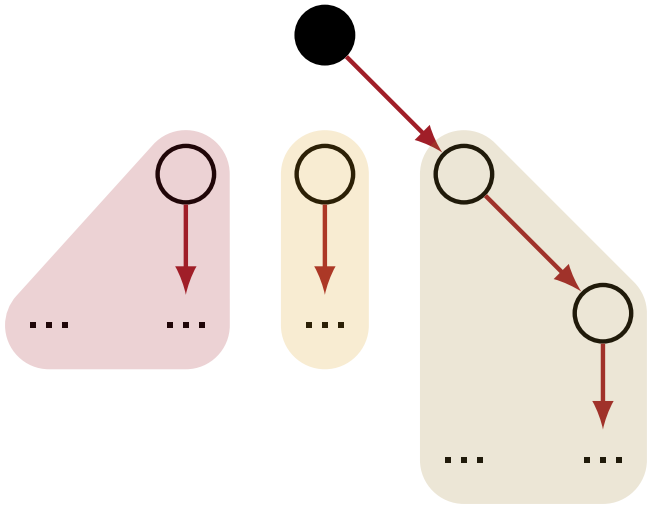
- Process tree RTGs in bottom-up fashion
- Extend table to track length of path starting at root

Tree RTGs



- Process tree RTGs in bottom-up fashion
- Extend table to track length of path (modulo 4) starting at root

Tree RTGs

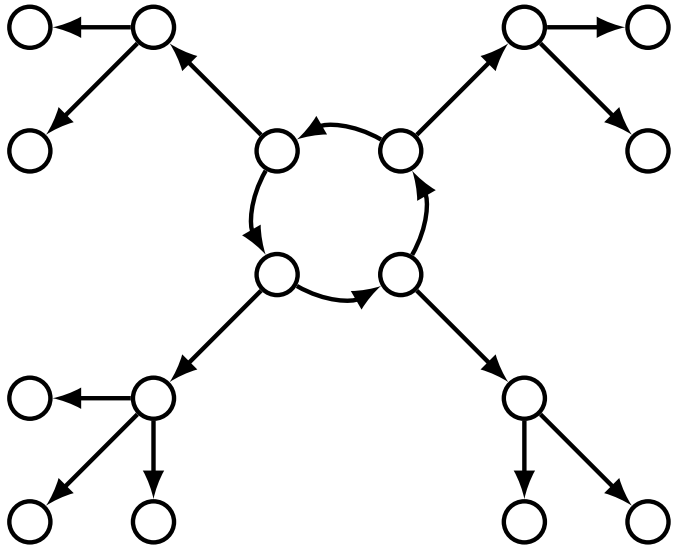


- Process tree RTGs in bottom-up fashion
- Extend table to track length of path (modulo 4) starting at root

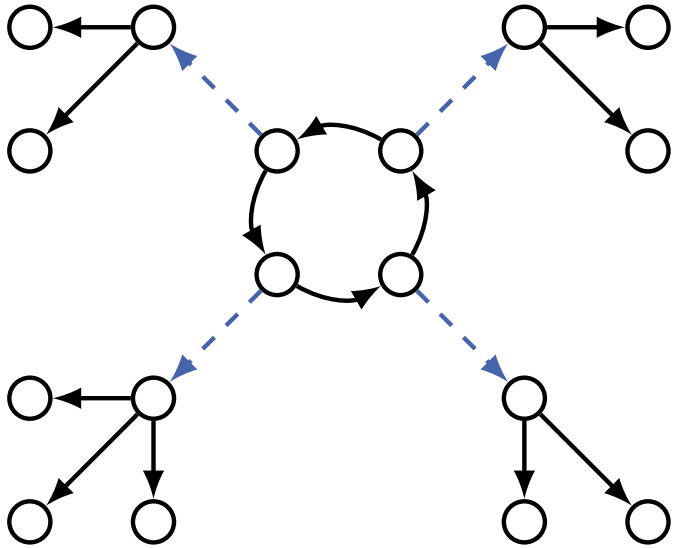
$\Rightarrow$ Can find optimal copy set C by trying all outgoing edges of root

$\Rightarrow$ Can be computed in time $O(n^3)$

Connected RTGs

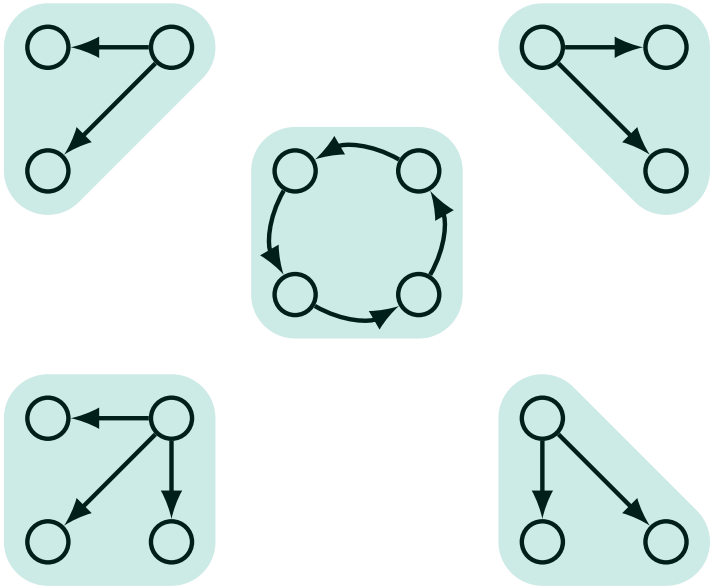


Connected RTGs



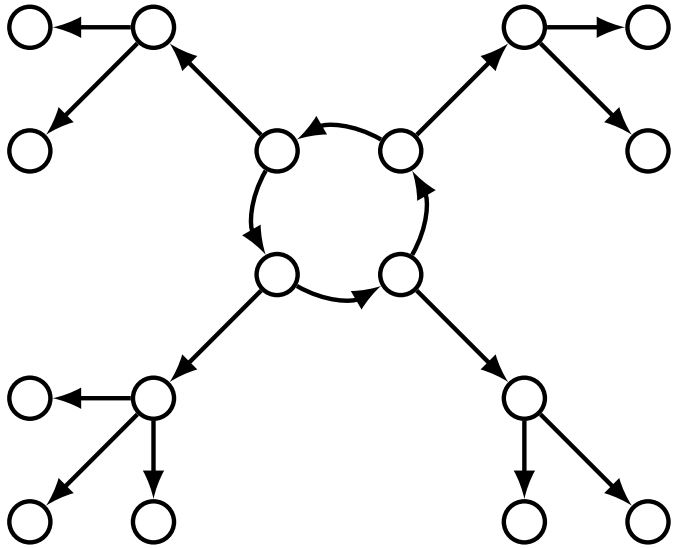
- Either keep cycle K and put edges leaving K into copy set

Connected RTGs



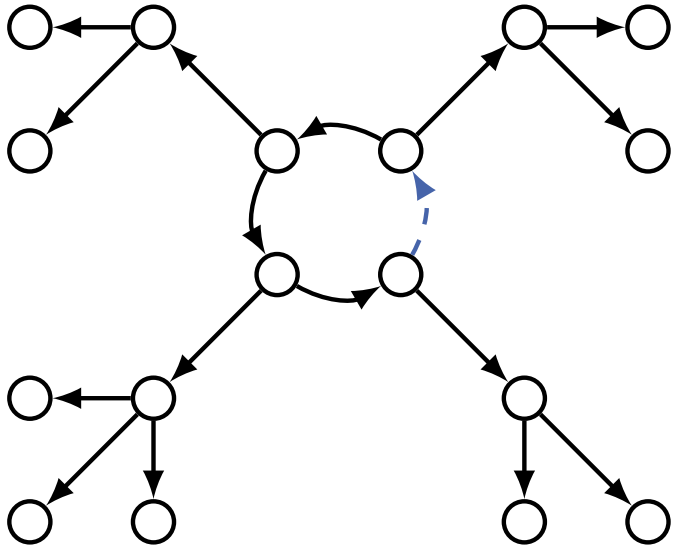
- Either keep cycle K and put edges leaving K into copy set

Connected RTGs



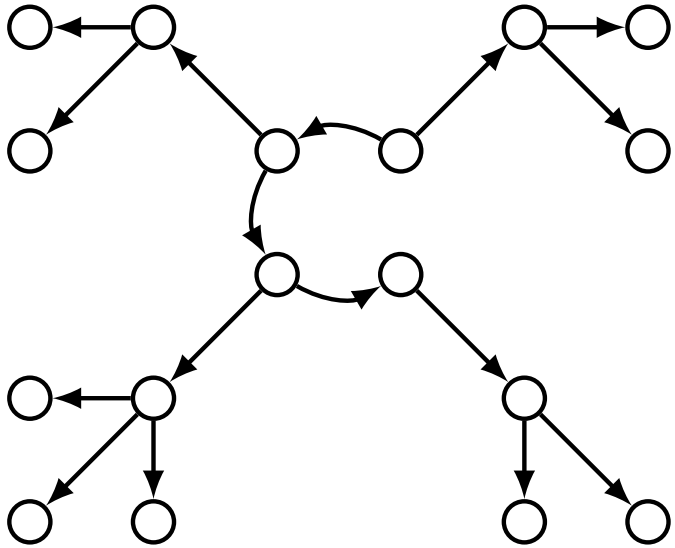
- Either keep cycle K and put edges leaving K into copy set

Connected RTGs



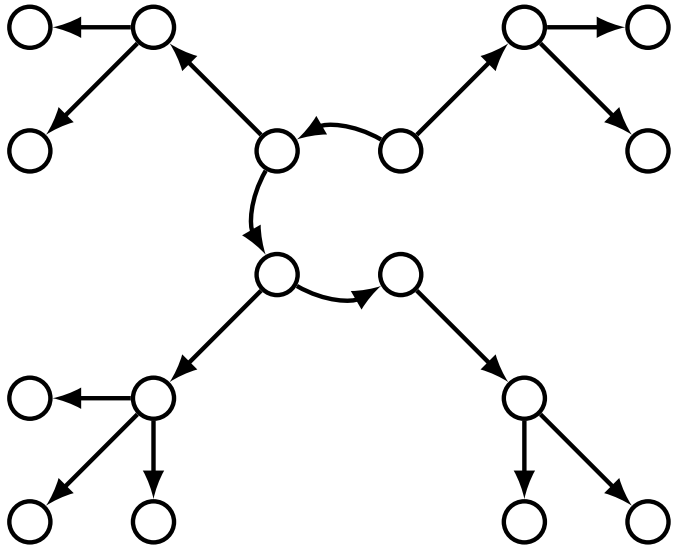
- Either keep cycle K and put edges leaving K into copy set
- Or cut K, $G - C$ is a tree

Connected RTGs



- Either keep cycle K and put edges leaving K into copy set
- Or cut K, $G - C$ is a tree

Connected RTGs



- Either keep cycle K and put edges leaving K into copy set
- Or cut K, $G - C$ is a tree

$\Rightarrow$ Can find optimal copy set C by trying all of K's edges (or keeping K)

$\Rightarrow$ Can be computed in time $O(n^4)$